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SUMMARY

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Equations are developed which show an analytical approach to simulating the 0-g impact loading of satellite swinging booms. The equations are obtained by simple manipulation of the equations of motion and the kinetic and potential energies involved and are verified by data from the actual spin tests of the International Ionosphere Satellite Ariel I (1962 o1).

AUTHOR

CONTENTS

LIST OF SYMBOLS

a Distance from the system spin axis to the boom pivot points.

E Energy contained in all appendages due only to the component of boom angular velocity parallel to the satellite spin axis, that is, $\dot{\theta}$.

I Total moment of inertia about the spin axis of the satellite, spent last stage, and attachments.

I_b Total moment of inertia about the spin axis of the satellite, spent last stage, and attachments minus the inertia of the booms.

J Total moment of inertia about the spin axis of the test model and attachments.

J_b Total moment of inertia about the spin axis of the test model and attachments minus the inertia of the booms.

L Total torque on the booms during erection.

l Arm length, or the distance from the boom pivot point to the boom weight m.

M Total mass of each uniformly distributed boom arm.

m Mass on the end of each boom.

n Number of booms on the satellite.

r Distance from the boom pivot point to the center of the boom weight m.

T Total energy of satellite, spent last stage, and attachments.

V Total potential energy of the spinning system in a 1-g conservative field.

V_θ Total potential energy of the test system for any value of θ.

W Total weight of each uniformly distributed boom arm.

w Weight on the end of each boom.

δ Total deflection at impact of the end of each appendage.

θ Boom angle from the folded position where $\theta = 0°$.

$\dot{\theta}$ First derivative of θ, or the component of the angular velocity of all appendages in a direction parallel to the spin axis.

$\dot{\lambda}$ Angular velocity of entire system about the intended spin axis.

$\ddot{\lambda}$ First derivative of $\dot{\lambda}$, or the angular acceleration of the entire system about its spin axis.

ρ Weight per unit length of the uniformly distributed boom arm; obtained by dividing the density of the boom arm by the cross-sectional area.

General Notation

Subscript $_0$: Initial system condition prior to boom erection.

Subscript $_2$: Final system condition after boom erection.

Subscript $_g$: Refers to an expression written for the 1-g force field condition.

Subscript $_z$: Refers to an expression written for the 0-g force field condition.

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INTRODUCTION

Satellite missions often dictate that the payload have appendages, or booms, extending out from the main structure. Because of physical limitations, it may be necessary to open the booms from a closed position after nose-fairing release. The booms usually will be erected either by a telescoping mechanism or, if space is limited in the main satellite body, by a swinging boom technique in which the folded booms swing out when released. Prior to the last-stage ignition normally the system will be spin stabilized and energy for the boom erection will result from the spin.

The main problem involved in developing an erectable swinging boom is to devise testing procedures which simulate actual erection conditions in a 0-g field.

This report gives a method used during development of the International Ionosphere Satellite Ariel I (1962 o1) for simulating boom erection conditions in a 0-g field.

ANALYSIS

Figure 1 is a sketch of the spin system and coordinate system which is discussed. The parameters are:

- a Distance from the system spin axis to the boom pivot points.
- x Distance from the boom pivot point to a point on the boom arm.

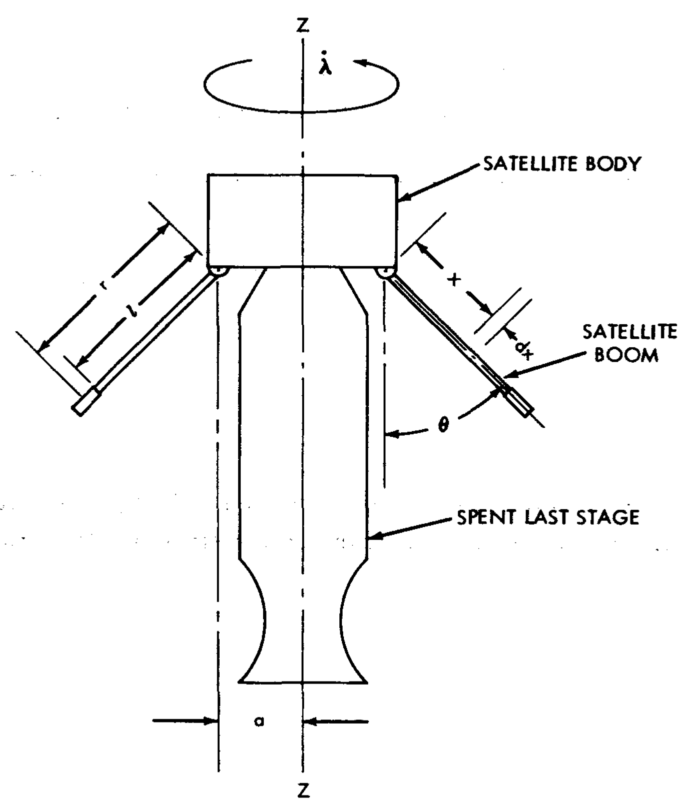


Figure 1—Satellite and last stage.

m Mass on the end of each boom.
l Arm length, or the distance from the boom pivot point to the boom weight m.
r Distance from the boom pivot point to the center of the boom weight m.
θ Boom angle from folded position where $\theta = 0°$.
$\dot{\lambda}$ Angular velocity of the entire system about the symmetrical axis.

It is assumed that there are two booms 180 degrees apart. The free swinging booms will normally be designed to take the following two types of loads:

1. The total torque L on the boom system during raising of the boom.
2. The energy E_z that must be absorbed to stop the booms in the erected position.

Normally at the time of a satellite boom erection the system is spinning essentially in a 0-g field and moving at a constant velocity; thus, the system can be examined analytically as if it were stationary and spinning. It is relatively easy to show that the total torque on the two boom hinges during swing-up is given by the expression

$$L = nmr\ddot{\lambda}(r \sin\theta + a) + 2nmr^2\dot{\lambda}\dot{\theta}\cos\theta + \frac{1}{3}nMl\ddot{\lambda}\left(l\sin\theta + \frac{3}{2}a\right) + \frac{2}{3}nMl^2\dot{\lambda}\dot{\theta}\cos\theta \quad , \tag{1}$$

where

n is the number of appendages,
M is the total mass of each boom arm, and
$\ddot{\lambda}$ is the system's angular acceleration about its spin axis.

The total kinetic energy of the whole satellite system is given by the expression

$$T_z = \frac{1}{2}nmr^2\dot{\theta}^2 + \frac{1}{2}n\rho\dot{\theta}^2\int_0^l x^2 dx + \frac{1}{2}nm\dot{\lambda}^2(r\sin\theta + a)^2$$

$$+ \frac{1}{2}n\rho\dot{\lambda}^2\int_0^l (x\sin\theta + a)^2\, dx + \frac{1}{2}I_b\dot{\lambda}^2 \quad , \tag{2}$$

where ρ is the uniformly distributed weight per unit length (lb/in) of the boom arm and I_b is the total moment of inertia of the entire structure less that of the booms. Equation 2 reduces to

$$T_z = \frac{1}{2}nmr^2\dot{\theta}^2 + \frac{1}{6}nMl^2\dot{\theta}^2 + \frac{1}{2}nm\dot{\lambda}^2(r\sin\theta + a)^2$$

$$+ \frac{1}{2}nM\dot{\lambda}^2\left(\frac{1}{3}l^2\sin^2\theta + al\sin\theta + a^2\right) + \frac{1}{2}I_b\dot{\lambda}^2 \quad . \tag{3}$$

Both momentum and energy are conserved in the system. In the Lagrange formulation, the partial derivative of Equation 3 with respect to $\dot{\lambda}$ gives the momentum equation for the entire system, since $\partial T/\partial\lambda = 0$;

$$\frac{\partial T}{\partial \dot{\lambda}} = nm\dot{\lambda}\,(r \sin\theta + a)^2 + nM\dot{\lambda}\left(\frac{1}{3} l^2 \sin^2\theta + al \sin\theta + a^2\right) + I_b\dot{\lambda} \ . \tag{4}$$

Since $\theta = 0°$ and $\lambda = \lambda_0$ at time $t = 0$, the momentum equation becomes

$$na^2\dot{\lambda}_0(m + M) + I_b\dot{\lambda}_0 = \text{Constant} \ . \tag{5}$$

From the conservation of momentum it follows that

$$\frac{\dot{\lambda}}{\dot{\lambda}_0} = \frac{na^2(m + M) + I_b}{nm\,(r \sin\theta + a)^2 + nM\left(\frac{1}{3} l^2 \sin^2\theta + al \sin\theta + a^2\right) + I_b} \ . \tag{6}$$

When Equation 6 is differentiated with respect to θ and multiplied by $\dot{\theta}$, an expression for $\ddot{\lambda}$ is obtained. With the expression for $\ddot{\lambda}$ and Equations 1 and 3, it is easy to plot the energy contained in the booms due to $\dot{\theta}$ and the torque on the hinges. If this procedure is followed, it will become obvious that *the energy due to* $\dot{\theta}$ *contained in the booms at full erection governs the structural design of the boom far more than does the torque during erection.* For acceptance testing of a boom systen it is therefore important to simulate the kinetic energy of the boom at impact in the erected position This paper presents a method for doing this and neglects any attempted simulation of the torsional forces involved.

SIMULATION PROCEDURE

To simplify calculations it is assumed that all tests will be run with the spin axis of the system parallel to the 1-g force field. Also, it is assumed that full erection occurs at $\theta = \pi/2$.

For a correct simulation at $\theta = \pi/2$, the kinetic energy E_g contained by the booms due to $\dot{\theta}$ when tested in a 1-g field must equal the kinetic energy E_z in the booms due to $\dot{\theta}$ when erection occurs in space, that is, $E_z = E_g$.

Solution for E_z

Because of the conservation of energy, T_z = constant. When T_z is evaluated at $t = 0$ $(\theta = 0°)$ and the result is equated to Equation 3, an expression is obtained which may be solved for E_z .

$$\begin{aligned} E_z &= \frac{1}{2} n\dot{\theta}^2\left(mr^2 + \frac{1}{3} Ml^2\right) \\ &= \frac{1}{2} na^2\dot{\lambda}_0^2 (m + M) + \frac{1}{2} I_b\dot{\lambda}_0^2 - \frac{1}{2} nm\dot{\lambda}^2 (r \sin\theta + a)^2 \\ &\quad - \frac{1}{2} nM\dot{\lambda}^2 \left(\frac{1}{3} l^2 \sin^2\theta + al \sin\theta + a^2\right) - \frac{1}{2} I_b\dot{\lambda}^2 \ . \end{aligned} \tag{7}$$

Evaluated at $\theta = \pi/2$, the expression is:

$$E_z = \frac{1}{2} na^2\dot{\lambda}_0^2 (m + M) + \frac{1}{2} I_b\dot{\lambda}_0^2$$

$$- \frac{1}{2} nm\dot{\lambda}_2^2 (r + a)^2 - \frac{1}{2} nM\dot{\lambda}_2^2 \left(\frac{1}{3} l^2 + al + a^2\right) - \frac{1}{2} I_b\dot{\lambda}_2^2 \quad , \tag{8}$$

which is easily simplified to

$$E_z = \frac{1}{2} I_0\dot{\lambda}_0^2 \left(1 - \frac{I_0}{I_2}\right) . \tag{9}$$

Solution for E_g

The term E_g is obtained by first writing Equation 3 for a system in a conservative field of 1-g, that is,

$$T_g = \frac{1}{2} nmr^2\dot{\theta}^2 + \frac{1}{6} nMl^2\dot{\theta}^2 + \frac{1}{2} nm\dot{\lambda}^2 (r \sin\theta + a)^2$$

$$+ \frac{1}{2} nM\dot{\lambda}^2 \left(\frac{1}{3} l^2 \sin^2\theta + al \sin\theta + a^2\right) + \frac{1}{2} J_b\dot{\lambda}^2 + V \quad , \tag{10}$$

where V equals the total potential energy of the entire system. At $t = 0$, $\theta = 0°$, this becomes

$$T_g = \frac{1}{2} na^2\dot{\lambda}_0^2 (m + M) + \frac{1}{2} J_b\dot{\lambda}_0^2 + V_0 = \text{Constant} . \tag{11}$$

This expression can now be substituted for T_g in Equation 10, and the result solved for E_g:

$$E_g = \frac{1}{2} n\dot{\theta}_2^2 \left(mr^2 + \frac{1}{3} Ml^2\right)$$

$$= \frac{1}{2} na^2\dot{\lambda}_0^2 (m + M) + \frac{1}{2} J_b\dot{\lambda}_0^2 + V_0 - \frac{1}{2} nm\dot{\lambda}^2 (r \sin\theta + a)^2$$

$$- \frac{1}{2} nM\dot{\lambda}^2 \left(\frac{1}{3} l^2 \sin^2\theta + al \sin\theta + a^2\right) - \frac{1}{2} J_b\lambda^2 - V_\theta \quad , \tag{12}$$

where V_θ denotes the potential energy of the entire system for any value of θ.

When Equation 12 is evaluated at $\theta = \pi/2$ and simplified, the following is obtained:

$$E_g = \frac{1}{2} J_0 \dot{\lambda}_{0g}^2 \left(1 - \frac{J_0}{J_2}\right) - \left(V_2 - V_0\right) , \tag{13}$$

where the term $(V_2 - V_0)$ expresses the change in potential energy of the entire system due to the erection of the booms.

Solution for $\dot{\lambda}_{0g}$

If the initial angular velocity $\dot{\lambda}_0$ and all of the inertias, masses, and dimensions for the entire system are the same in both the 0-g and 1-g states, then by comparing Equations 9 and 13 it can be seen what must differ between the two conditions. Since in a 1-g field the term $(V_2 - V_0)$ is not zero, E_g will be less than E_z by the amount $(V_2 - V_0)$. The boom angular velocity $\dot{\theta}_{2g}$ at $\theta = \pi/2$ in the 1-g field must be less than $\dot{\theta}_{2z}$ in the 0-g field. Therefore, to achieve the result $E_z = E_g$, the right-hand sides of Equations 9 and 13 must be made equal at the point of full erection. Thus,

$$\frac{1}{2} I_0 \dot{\lambda}_0^2 \left(1 - \frac{I_0}{I_2}\right) = \frac{1}{2} J_0 \dot{\lambda}_{0g}^2 \left(1 - \frac{J_0}{J_2}\right) - \left(V_2 - V_0\right) . \tag{14}$$

Inspection of Equation 14 shows that the inertias, the initial system spin rate, or both may be altered to make the identity hold. Since normally any testing will be accomplished with a model of the satellite, it usually is easy to "slug up" the system so that $I_0 = J_0$ and $I_2 = J_2$. Therefore, with a known initial design spin rate of $\dot{\lambda}_0$ it is easy to solve Equation 14 for the only unknown, the initial spin rate of the test system $\dot{\lambda}_{0g}$:

$$\dot{\lambda}_{0g} = \sqrt{\dot{\lambda}_0^2 + \frac{2I_2 \left(V_2 - V_0\right)}{I_0 \left(I_2 - I_0\right)}} . \tag{15}$$

Test Arrangement

The spin test setup should be a simulation of the hardware shown in Figure 1 mounted on a suitable spin table. A typical setup is shown in Figure 2.

It is imperative that the entire system be "free-wheeling" during the boom erection, that is, that there be as little as possible externally applied torque (drive power, friction, air drag, etc.) acting on the system. Some type of clutching arrangement, as is shown in Figure 2, is

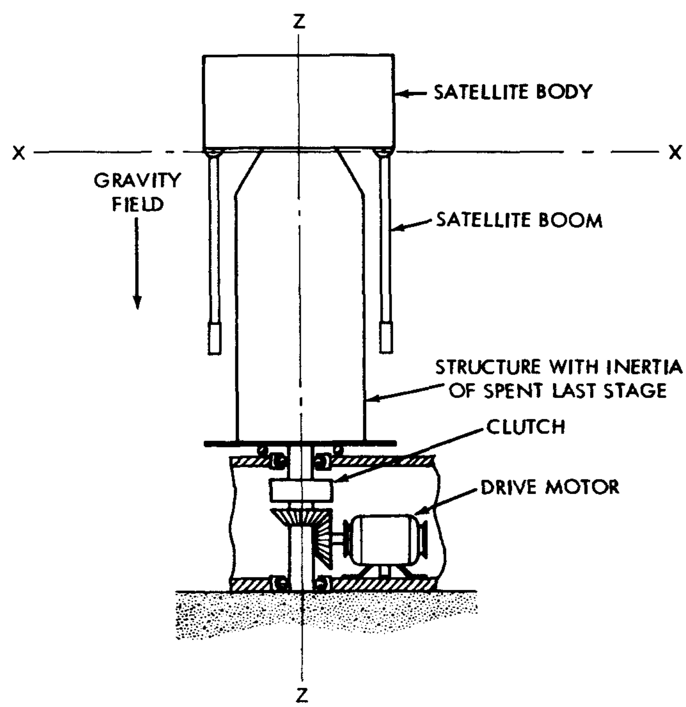


Figure 2–Spin test setup.

therefore necessary to remove the drive motor from the system just prior to boom release. A well engineered bearing design will reduce the friction torque to a negligible amount. The torque induced by wind resistance generally may be neglected unless the swinging appendages present a large area; in such a case it might be necessary to conduct the tests in a vacuum chamber.

Use of $\dot{\lambda}_{0g}$

With the setup shown in Figure 2, the only mass to undergo a potential energy change will be the swinging booms.

By solving for $V_2 - V_0$ and applying this to Equation 15 for an actual initial spin rate of $\dot{\lambda}_0$, a new initial spin rate $\dot{\lambda}_{0g}$ is obtained. If in a force field test the booms are released at a system angular velocity of $\dot{\lambda}_{0g}$, the booms' angular velocity $\dot{\theta}$ will equal the 0-g boom angular velocity at one point, that is, at $\theta = \pi/2$, and the simulation will be sufficient.

If the booms are swung in the up direction during the tests, then $\dot{\lambda}_{0g}$ will be higher than the actual flight $\dot{\lambda}_0$. Also, because of the conservation of momentum, the final test angular velocity $\dot{\lambda}_{2g}$ naturally will be higher than the actual flight $\dot{\lambda}_2$. The change in system energy during erection in the tests will be greater than in space flight by the amount of potential energy gained; however, the angular velocity of the booms will be flight level at the instant of erection.

It should be remembered that the tests may be run with the appendages swinging down rather than up. In this case the potential energy to be dissipated by the booms would have to be subtracted from the initial system energy.

The procedure described, with the appendages swinging up, was used for acceptance tests of the satellite Ariel I. The solar paddles and the mass booms, which were used for spin stabilization, were tested in this manner. Appendix A gives the dynamic calculations for simulated flight erections of the mass booms.

CONCLUSION

This method of testing has distinct advantages: it can be quickly incorporated into a test program; no uncertain quantities are introduced into the test setup; and no elaborate test setup is necessary.

Examination of the equations involved will reveal the disadvantages or limitations. It is always necessary to know accurately and to control all parameters (spin rates, inertias, and energy changes) in the test system. If the change in potential energy $(V_2 - V_0)$ is large in comparison to the planned initial spin rate $\dot{\lambda}_{0z}$, then precision control of all test parameters becomes necessary to prevent the introduction of a large simulation error. Correctly used, however, this technique provides a proper means of simulating the space erection of satellite appendages, and tests the entire system for impact.

Appendix A

Calculations for Simulating the Zero-Gravity Erection of the Ariel I Mass Booms

As is shown in Figure A1, the Ariel I satellite had eight erectable appendages: two experiment-carrying booms, two mass booms for spin stabilization, two solar paddles with rigid arms, and two solar paddles with secondary hinges in the arms. In actual flight, prior to any erection, the appendages were folded down beside the last-stage rocket bottle. The planned erection sequence was as follows: (1) the system was to be spin-stabilized at 161 rpm prior to the last-stage ignition; (2) 15 minutes after last-stage ignition, a stretch yo-yo de-spin device would unwind, reducing the spin rate to 76 rpm; (3) 60 seconds after yo-yo release the two experimental booms

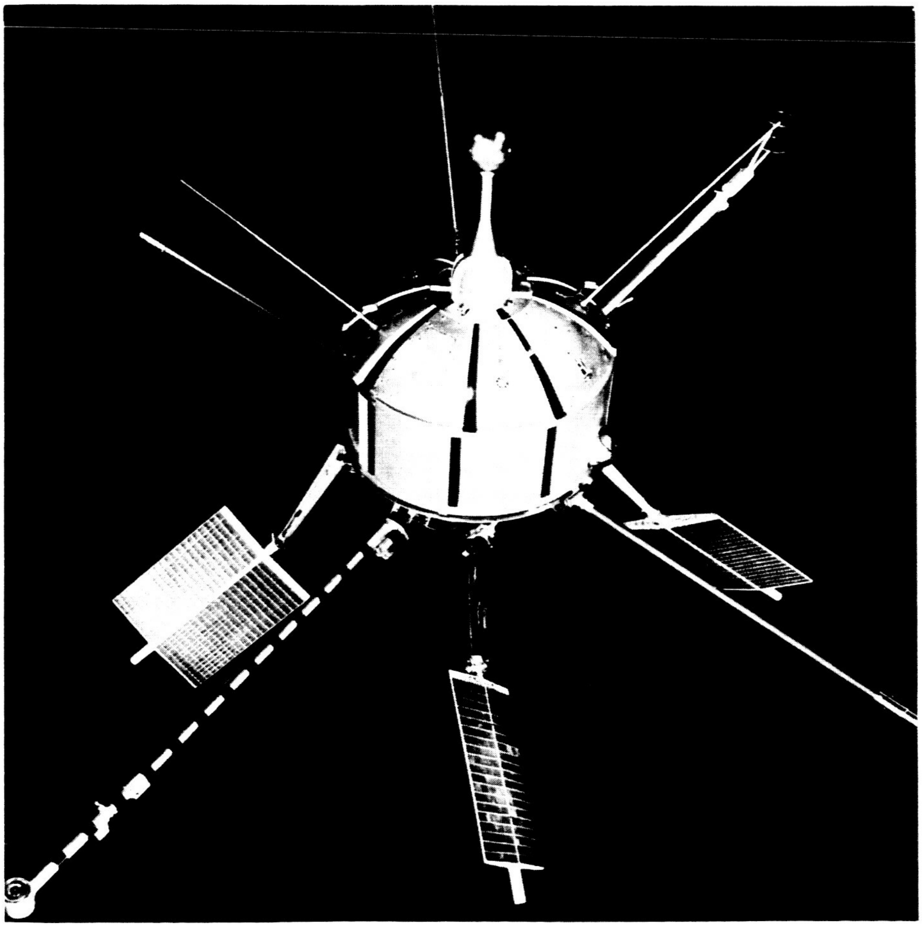


Figure A1—The Ariel I satellite.

were to be released and, under the constraint of a governor, erect in approximately 3.0 seconds; (4) 60 seconds after the previous action, with the system at approximately 52 rpm, the four solar paddles and two mass booms would be released simultaneously and allowed to swing freely to their erected positions. At this point the system would be at approximately 35 rpm; (5) 60 seconds later, separation of the spacecraft from the last-stage bottle would occur and the satellite would be in orbit as seen in Figure A1.

In the design of the solar paddles and mass booms it was necessary to solve for the impact energy at erection of each type appendage. Since they were all released at the same instant but did not all reach their erected positions at the same instant, the analytical solution of the process was considerably more involved than the case where only one type of appendage erects at a time. In the last case, the impact energy at erection in a force free field is:

$$E_z = \frac{1}{2} I_0 \dot{\lambda}_0^2 \left(1 - \frac{I_0}{I_2}\right) , \tag{A1}$$

where

E_z is the energy that must be absorbed to stop the booms in the erected position,

I_0 is the initial total moment of inertia about the spin axis of the satellite, spent last stage, and attachments,

I_2 is the final total moment of inertia about the spin axis of the satellite, spent last stage, and attachments,

$\dot{\lambda}_0$ is the initial angular velocity of the entire system about the symmetrical axis.

The more involved and actual process of simultaneous erection of dissimilar appendages was solved by writing Lagrangian expressions involving the three types of appendages and solving the equations on a computer. The resultant actual impact energy load on the two mass booms due to a flight erection of the six appendages at an initial $\dot{\lambda}_{0z}$ of 52 rpm was calculated to be 8.51 ft-lb. During development testing of the appendages, it was necessary to test the solar paddles and mass booms separately. For a proper acceptance test of the mass booms it was then necessary to conduct tests simulating the booms containing 8.51 ft-lb of kinetic energy at the point of erection. When the mass booms were tested separately, this was accomplished by first calculating an initial 0-g system angular velocity $\dot{\lambda}_{0z}$, which corresponds to 8.51 ft-lb of kinetic energy in the mass booms at erection. For the tests it was assumed the experimental booms were open prior to the mass booms release and the solar paddles remained folded. The Ariel I moments of inertia for these conditions were: I_0, 4.01 slug-ft^2; and I_2 after mass boom full erection, 4.84 slug-ft^2. With these values, Equation A1 gives $\dot{\lambda}_{0z}$ = 47.5 rpm. The test, therefore, was to simulate a 0-g mass boom erection at an initial rotation rate of 47.5 rpm.

From Equation 15 in the body of the report, that is,

$$\dot{\lambda}_{0g} = \sqrt{\dot{\lambda}_0^2 + \frac{2I_2 (V_2 - V_0)}{I_0 (I_2 - I_0)}} , \tag{A2}$$

an initial spin rate λ_{0_g} was calculated to simulate a space erection at 47.5 rpm. Figures A2 and A3 show the test system before and after erection.

To use Equation A2, the potential energy change $V_2 - V_0$ between the configurations shown in Figures A2 and A3 was calculated. Obviously, the mass booms were the only consideration. Figure A4 shows the extreme positions of each mass boom.

The weight w on the end of each arm was 0.775 lb. For $E_z = 8.51$ ft-lb the deflection δ was calculated to be approximately 5.9 inches. This δ was large enough to warrant consideration when calculating $V_2 - V_0$. Therefore

$$V_2 - V_0 = n\rho \int_0^l (x - x \cos\theta)\, dx + nw \int_0^r (1 - \cos\theta)\, dx + n\rho \int_0^l z dx + nw \int_0^\delta dz \, , \qquad \text{(A3)}$$

where the last two terms in the expression consider the deflection. For full erection at $\theta = \pi/2$ and with $l \approx 34.0$ (arm extended into the weight), the first two terms of Equation A3 are:

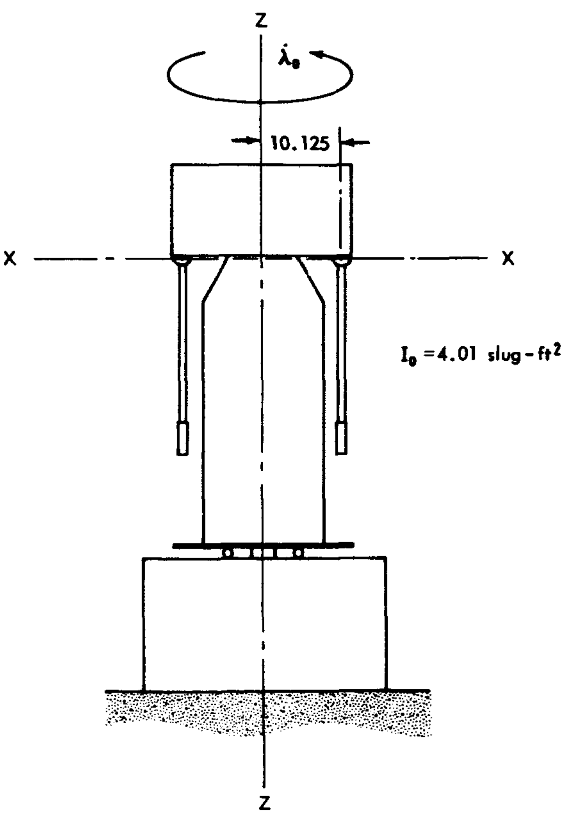


Figure A2–Test system before boom erection.

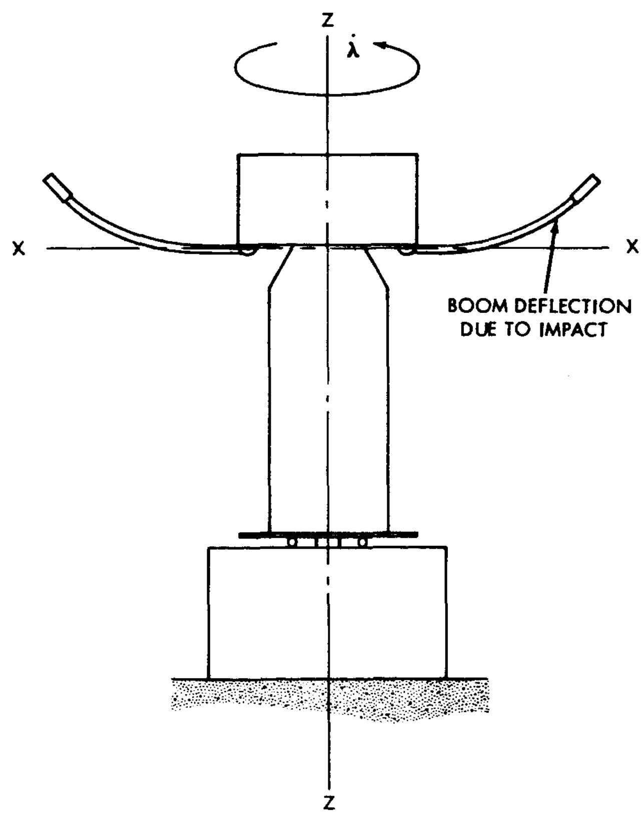


Figure A3–Test system at boom erection.

$$n\rho \int_0^l (x - x\cos\theta)dx + nw \int_0^r (1 - \cos\theta)dx = n\rho \frac{l^2}{2} + nwr$$

$$= 77.3 \text{ in-lb} .$$

A close approximation for the third term in Equation A3 was obtained by approximating the curve of the deflected boom arm in Figure A4 by a quarter cosine wave function*, that is,

$$z = \delta\left(1 - \cos \frac{\pi x}{2l}\right)$$

and

$$n\rho \int_0^l z dx = n\rho\delta \int_0^l \left(1 - \cos \frac{\pi x}{2l}\right) dx$$

$$= n\rho\delta l \left(1 - \frac{2}{\pi}\right) = 3.2 \text{ in-lb} . \quad \text{(A4)}$$

The last term in Equation A3 is simply:

$$nw \int_0^\delta dz = 9.0 \text{ in-lb.} , \quad \text{(A5)}$$

and so the total potential energy change is

$$V_2 - V_0 = 77.3 + 3.2 + 9.0 = 89.5 \text{ in-lb}$$

$$= 7.46 \text{ ft-lb} .$$

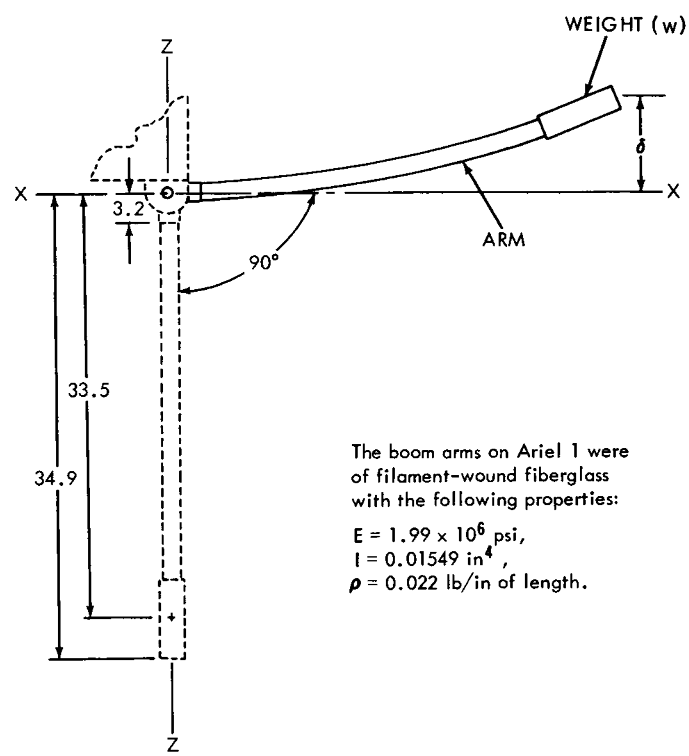
All the known values are then applied to Equation A2, and

$$\dot{\lambda}_{0g} = \sqrt{(4.974)^2 + \frac{2(4.84)(7.46)}{4.01(4.84 - 4.01)}}$$

$$= 65.1 \text{ rpm} .$$

Thus, the actual configuration shown in Figure A5 was spun up to approximately 67.0 rpm, the drive disengaged, and the spin

*Den Hartog, J. P., "Mechanical Vibrations," 4th Ed., New York: McGraw-Hill, 1956, p. 153.



The boom arms on Ariel 1 were of filament-wound fiberglass with the following properties:

$E = 1.99 \times 10^6$ psi,
$I = 0.01549$ in^4,
$\rho = 0.022$ lb/in of length.

Figure A4—Mass boom displacement.

allowed to decay to 65.1 rpm, at which point the booms were released and allowed to swing freely to the erected position. Since deflection was considered, $\dot{\theta}_g$ was slightly higher than the actual flight $\dot{\theta}_z$ at $\theta = \pi/2$. However, the total potential energy in the booms at full deflection for the tests, when $\dot{\lambda}_{0g} = 65.1$ rpm, equaled the potential energy in the booms at full deflection for a space erection when $\dot{\lambda}_{0z} = 47.5$ rpm. Figure A6 shows the calculated plots of the kinetic energy in the booms versus the boom angle θ for the actual space and test conditions.

From Figure A6 it is seen that at the instant when $\theta = 90°$, or full erection, the booms had about 12 in-lb more kinetic energy in the tests than during the actual flight. However, the change in potential energy in the tests due to the upward deflection of the booms subtracted from the kinetic energy. From Equations A4 and A5 it will be seen that the calculated potential energy change due to deflection equals 12.2 in-lb. From high-speed motion pictures of the erection tests, it was ascertained that the total deflection δ of each boom end was approximately 6 inches. Therefore, at full deflection, the strain of each boom was approximately equal to that anticipated in the actual flight and simulation was completed.

If the erecting appendages had been fairly rigid and the deflection was negligible, then a graph of the variables in Figure A6 would have shown that the two kinetic energy curves terminate on the same value at $\theta = 90°$. The angular velocities $\dot{\theta}_z$ and $\dot{\theta}_g$ would have been equal at that point.

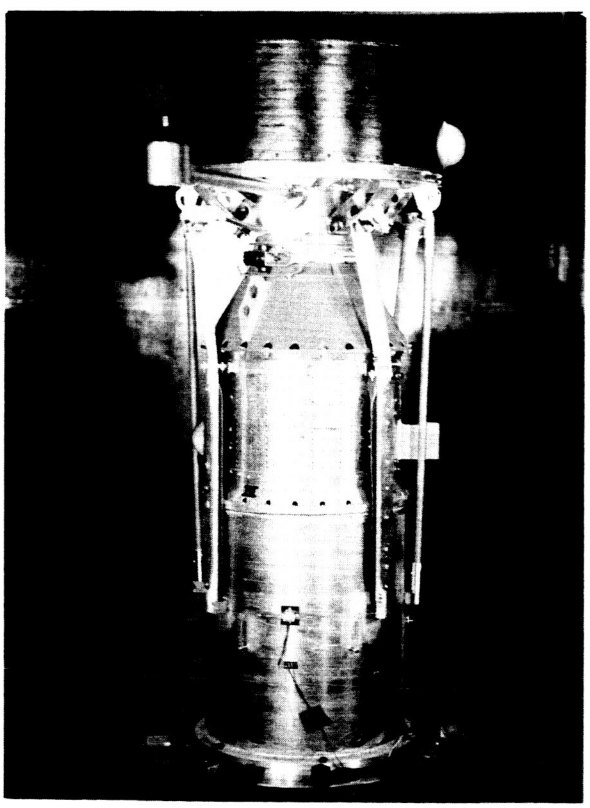


Figure A5—The Ariel I test configuration.

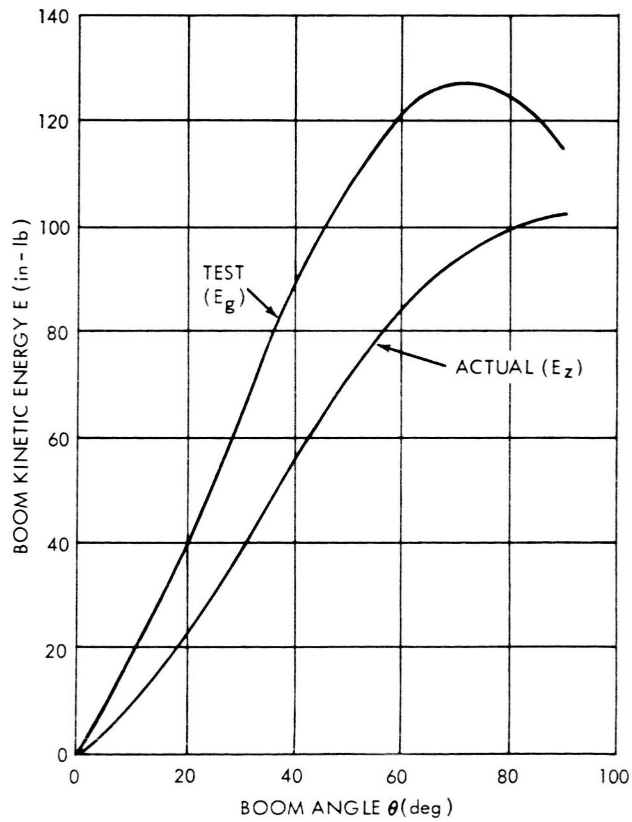


Figure A6—Calculated plots of the kinetic energy in the Ariel I mass booms under both test and actual flight conditions.